Tight Bounds on the Relative Performance of Pricing Mechanisms in Storable Good Markets

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Storable Good Market

- Selling over multiple time periods, agents have a demand at each time.

- **Goods are *storable* at a cost:**

  If a good gives value v at time t, and is bought at time $s < t$ for price p, then the buyer gets value $v - p - c \cdot (t - s)$, paying price *and* storage cost.

- **Restaurant example:**

  Restaurant sells a yogurt dip.

  Price of yogurt changes every day, but expensive to store.

  When to buy? Buy early and store?

Storable Goods Monopolist

- Wants to maximize revenue. Two pricing models:
  - **Pre-Announced Pricing** (Commiting ahead of time) Π^{PA}
  - **Contingent Pricing** (No-Commitment / Threat / Subgame Perfect) Π^{CP}
  - Computing optimal prices are very different optimization problems!

Question: Which Strategy gives more revenue?

- Economic Effects at Play:
  - **Consumption Effect:** Raising prices means fewer purchases
  - **Stockpiling Effect:** Raising prices causes more storage, and earlier purchase

Storage is *lost revenue*, because they were willing to pay

Example for Commit > Threat

Suppose storage cost $c = 1$
and consider demand as follows:

Consumer	$t = 1$	$t = 2$
Alice	$v = 1$	No demand
Bob	No demand	$v = 2$

- **Contingent Pricing:** $\Pi^{CP} = 2$

  It is optimal to set $p_2 = 2$, since subgame perfect.

  If $p_1 > 1$, Alice doesn't buy and Bob waits, so **Rev = 2**

  If $p_1 \leq 1$, Alice buys, and Bob buys early and stores, so **Rev = 2** p_1.

- **Pre-Announced Pricing:** $\Pi^{PA} = 2.999$

  Set $p_1 = 1$, $p_2 = 1.999$.

  Alice buys, Bob benefits from waiting instead of storing.

Example for Threat > Commit

Suppose storage cost $c = 1$
and consider demand as follows:

Consumer	$t = 1$	$t = 2$
Alice	$v = 6$	$v = 6$
Bob	$v = 4$	$v = 2$

- **Pre-Announced Pricing:** $\Pi^{PA} = 12$

  Set $p_1 = 4$, $p_2 = 2$, everyone buys. Or $p_1 = 6$, $p_2 = 6$, Alice buys.

- **Contingent Pricing:** $\Pi^{CP} = 14$

  If Alice doesn't buy early and store, monopolist sets $p_2 = 6$!

  So set $p_1 = 4$, Alice buys 2 units, and Bob buys 1,

  Then can set $p_2 = 2$ which gives **Rev = 14**.

Results

- Deciding which pricing strategy to use is not straightforward!

- **This paper:**
  Let N buyers and T time steps, then at any subgame-perfect equilibria,
  $\Pi^{PA} \leq \Pi^{CP} \cdot O(\log N + \log T)$, and this bound is tight

- Past work [Berbeglia, Rayaprolu, Vetta]:
  For any SPNE, $\Pi^{CP} \leq \Pi^{PA} \cdot O(\log N + \log T)$, and this bound is tight

- Coase conjecture (1972): Commitment always better than contingent.
  - Proved true in *infinite horizon*, not true in general. [Gul et al., 1986]

Proof of Upper Bound : $\Pi^{PA} \leq \Pi^{CP} \cdot O(\log N + \log T)$

- Introduce *fixed* price: commiting to a single price at all times. " Π^F "

- Clearly, $\Pi^F \leq \Pi^{PA}$, but, $\Pi^{PA} \leq \Sigma v_i \leq \Pi^F \cdot O(\log N + \log T)$,
  - If v^* is i-th best value, then j-th best value is at most $(i/j)\, v^*$
  - So $\Sigma v_i \leq v^* \,\Sigma(i/j) \leq \Pi^F \cdot ln(NT)$

- Want to show $\Pi^F \leq \Pi^{CP}$,

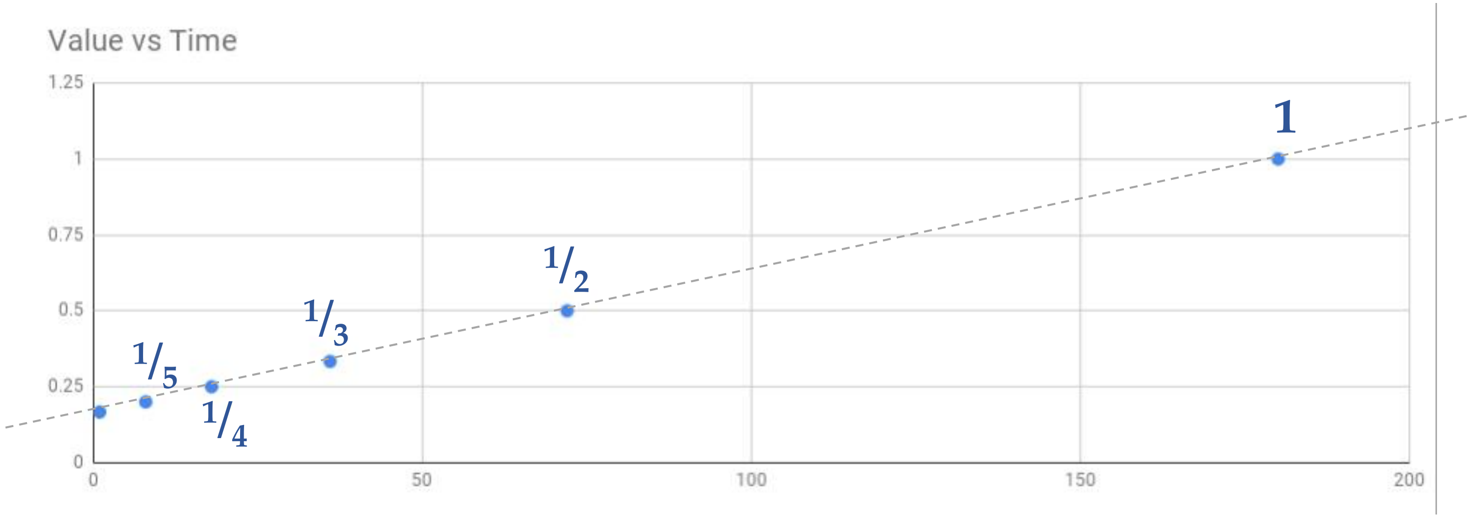
  Prove by showing that charging the optimal fixed price is suboptimal in the contingent setting, but sells at least as much as Π^F.

□

Tight Example

- Idea: n consumers, each demanding at different times,
- Consumer i has value $1/i$ for one unit, lower indices consume earlier
- Storage cost $\approx 1/n^3$
- Consumption times are spaced out just right so that, no one stores if

$$p_i = 1/i - (n - i + 1)/n^3$$



Tight Example

- No storage if $p_i = 1/i - O(1/n^2)$ but storage is better if $p_i = 1/i$

- Total commitment revenue is $\approx \log(n) - 1/n$
    - everyone buys at almost their total value

- However, contingent pricing equilibrium sells all-or-nothing at any time! So if you sell at time i, you get $i \cdot 1/i = 1$ in total, and the game ends
- Seller's strategy is to charge price $1/i$ when buyer i has value
  Buyer's strategy is to only buy if *someone* has value, and price is good enough
- Seller cannot improve: charging less loses revenue and everyone still buys, charging more leaves some revenue on the table that will disappear in the next round.
  - Buyer cannot improve since buying earlier means more savings

Thank you